AN EXACT DESIGN TECHNIQUE FOR A TYPE OF MAXIMALLY-FLAT QUARTER-WAVE-COUPLED BAND PASS FILTER

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Consider the filter configuration consisting of a number of shorted quarter wave stubs spaced a quarter wavelength apart on an otherwise uniform transmission line. What tapering of the characteristic impedances of the stubs will yield a maximally-flat transmission characteristic?

The insertion loss of a lossless, linear, passive, reciprocal and symmetrical two port network characterized by the general circuit parameters, ABCD, inserted between a generator and load each having an impedance of unity is given by the relation:

$$\frac{P_o}{P_L} = 1 - \frac{(B_n - C_n)^2}{4} \tag{1}$$

where B_n and C_n are determined by the matrices:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_n = [S_n]\,[L_{n-1}]\,[S_{n-1}]\,\ldots\,[L_2]\,[S_2]\,[L_1]\,[S_1] \tag{2}$$

where S_r is the ABCD matrix for the r^{th} stub and L_r is the ABCD matrix for the r^{th} connecting line. For the shorted shunt stubs whose normalized characteristic admittance is k_r:

$$[S_r] = \begin{bmatrix} 1 & 0 \\ k_r q & 1 \end{bmatrix} \tag{3}$$

where $q = -j \cot \theta$

$\theta = 2\pi \ell / \lambda$

ℓ is the length of the shorted stub.

For the connecting lines, whose length is the same as the stub length and whose characteristic admittance is unity:

$$[L_r] = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} = j \sin\theta \begin{bmatrix} q & 1 \\ 1 & q \end{bmatrix} \qquad (4)$$

The complete matrix for the filter then becomes:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_n = (j \sin\theta)^{n-1} \begin{bmatrix} 1 & 0 \\ k_n q & 1 \end{bmatrix} \begin{bmatrix} q & 1 \\ 1 & q \end{bmatrix} \cdots \begin{bmatrix} q & 1 \\ 1 & q \end{bmatrix} \begin{bmatrix} 1 & 0 \\ k_1 q & 1 \end{bmatrix} \qquad (5)$$

From which we have:

$$\begin{aligned} A_n &= (j \sin\theta)^{n-1} (f_a(q)) \\ B_n &= (j \sin\theta)^{n-1} (f_b(q)) \\ C_n &= (j \sin\theta)^{n-1} (f_c(q)) \\ D_n &= (j \sin\theta)^{n-1} (f_d(q)) \end{aligned} \qquad (6)$$

where the functions $f(q)$ are finite power series in q with functions of k_r as coefficients.

Hence $B_n - C_n$ contains of finite power series in q. To be maximally flat, the coefficients are all set equal to zero except that of the highest power of q. These equations, involving the k_r's, are then solved simultaneously to obtain the proper tapering of the characteristic admittances of the stubs.

It can be shown that in general, if the maximally flat conditions are imposed,

$$\frac{P_o}{P_L} = 1 + K_n \frac{\cos^{2n} \theta}{\sin^2 \theta} \tag{7}$$

where n is the number of stubs and

$$K_n = \left(\frac{k_1(k_2+2) \ \dots \ (k_n+2)}{2}\right)^2 \tag{8}$$

The following tables give some values for K_n and k_r which meet the requirements:

Three Stub Filter

10 log K_3	k_1	k_2
-12.728	0.100	0.200
- 0.944	0.300	0.600
+ 5.46	0.500	1.000
+10.138	0.700	1.400
+15.56	1.000	2.000
+21.156	1.400	2.800
+27.604	2.000	4.000
+31.904	2.5	5.0
+35.563	3.0	6.0

Four Stub Filter

10 log K_4	k_1	k_2
- 5.17	0.1	0.292
+ 3.253	0.2	0.571
+13.329	0.4	1.109
+25.668	0.8	2.141
+35.909	1.3	3.395
+44.873	1.9	4.877
+56.734	3.0	7.568

Five Stub Filter

$10 \log K_5$	k_1	k_2	k_3
+ 3.452	0.100	0.366	0.532
13.577	0.200	0.694	0.989
20.523	0.300	1.005	1.410
26.002	0.400	1.304	1.808
30.601	0.500	1.596	2.193
38.16	0.700	2.166	2.933
44.324	0.900	2.724	3.648
54.172	1.300	3.819	5.038
66.970	2.000	5.702	7.403
77.874	2.800	7.829	10.058

Six Stub Filter

$10 \log K_6$	k_1	k_2	k_3
+13.378	.100	.419	0.755
25.469	.200	.774	1.329
33.805	.300	1.105	1.838
40.388	.400	1.422	2.314
50.721	.600	2.031	3.207
58.863	.800	2.622	4.055
65.668	1.0	3.202	4.878
76.755	1.4	4.343	6.478
85.687	1.8	5.468	8.045
96.571	2.4	7.141	10.359

By way of experimental confirmation of the design technique, my associate, Mr. C. E. Becraft designed, built and tested a six element filter in coaxial line. The observed and calculated curves are compared in Figure I, the agreement is gratifying indeed in this filter whose bandwidth is greater than 70%.

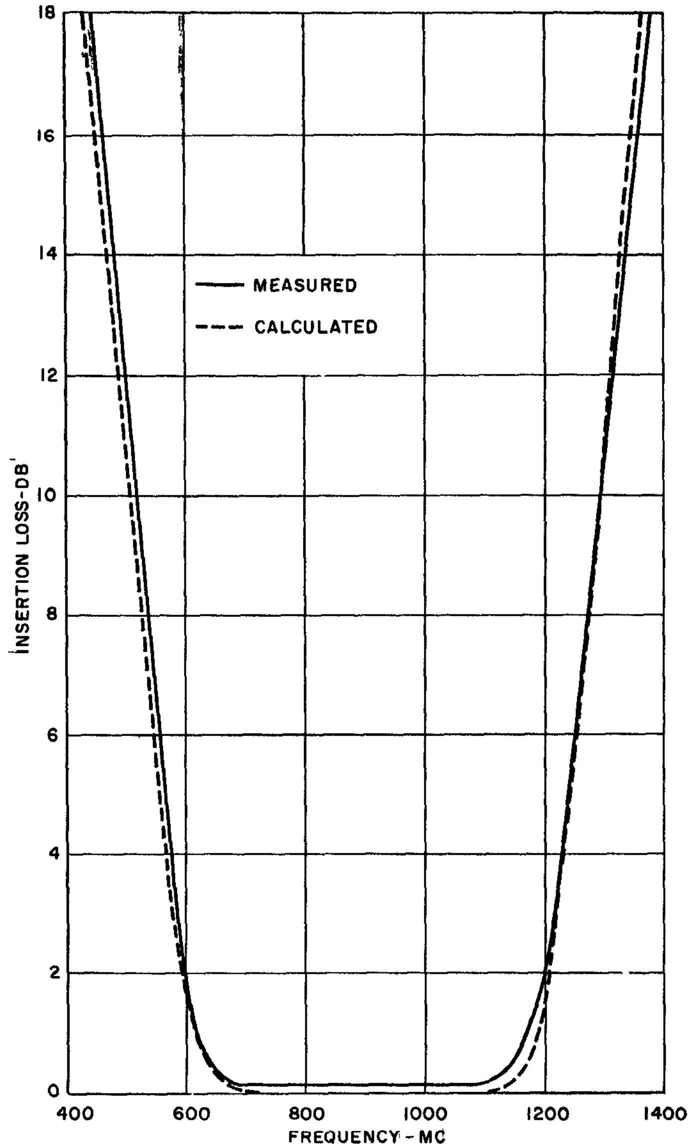

FIGURE I

Fig. I. Calculated and measured insertion loss of six-stub filter.

NOTES